외판원 문제(TSP)를 위한 보로노이 기반 발견적 해법

A Heuristic Algorithm Based On Voronoi Diagram for Traveling Salesman Problem

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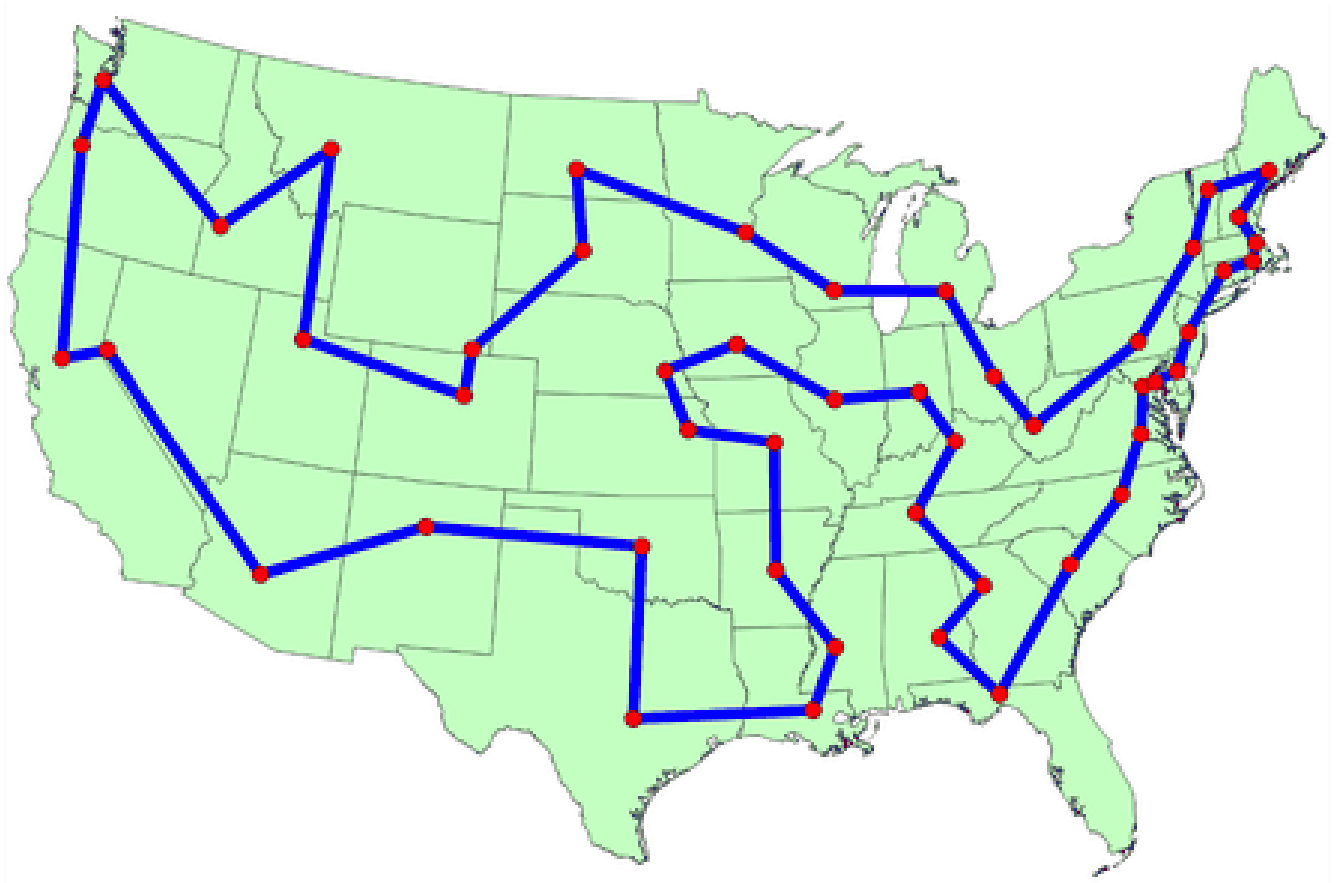
Traveling Salesman Problem

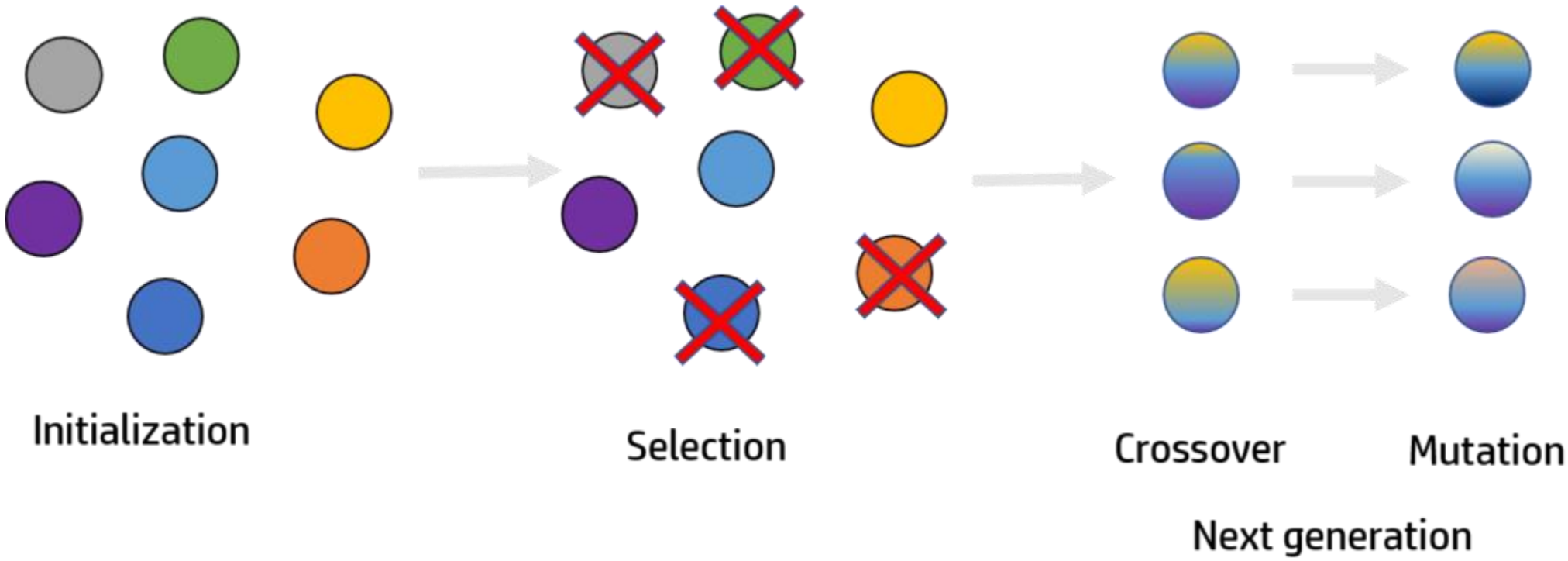
모든 도시를 한 번씩만 거쳐 시작지점으로 돌아오는 경로 중 최소 비용을 찾는 문제



https://optimization.cbe.cornell.edu/index.php?title=Traveling_salesman_problem

Genetic Algorithm

최적화 문제를 푸는 휴리스틱 기법
생물체가 환경에 적응하며 진화하는 모습을 모방하여 최적해를 찾아내는 방법



https://www.kaggle.com/code/marlesson/genetic-algorithm-with-deap

Approximation Algorithms

최적화 문제에서 $d(H^*)$의 p배를 벗어나지 않는 근사해를 보장하는 Algorithm

2-Approximation Algorithm

Minimum Spanning Tree

↓

Euler Circuit

↓

Hamiltonian Circuit

$$d(H) \leq 2d(H^*)$$

1.5-Approximation Algorithm
(Christofides Algorithm)

Minimum Spanning Tree

↓

Minimum Perfect Matching

↓

Euler Circuit

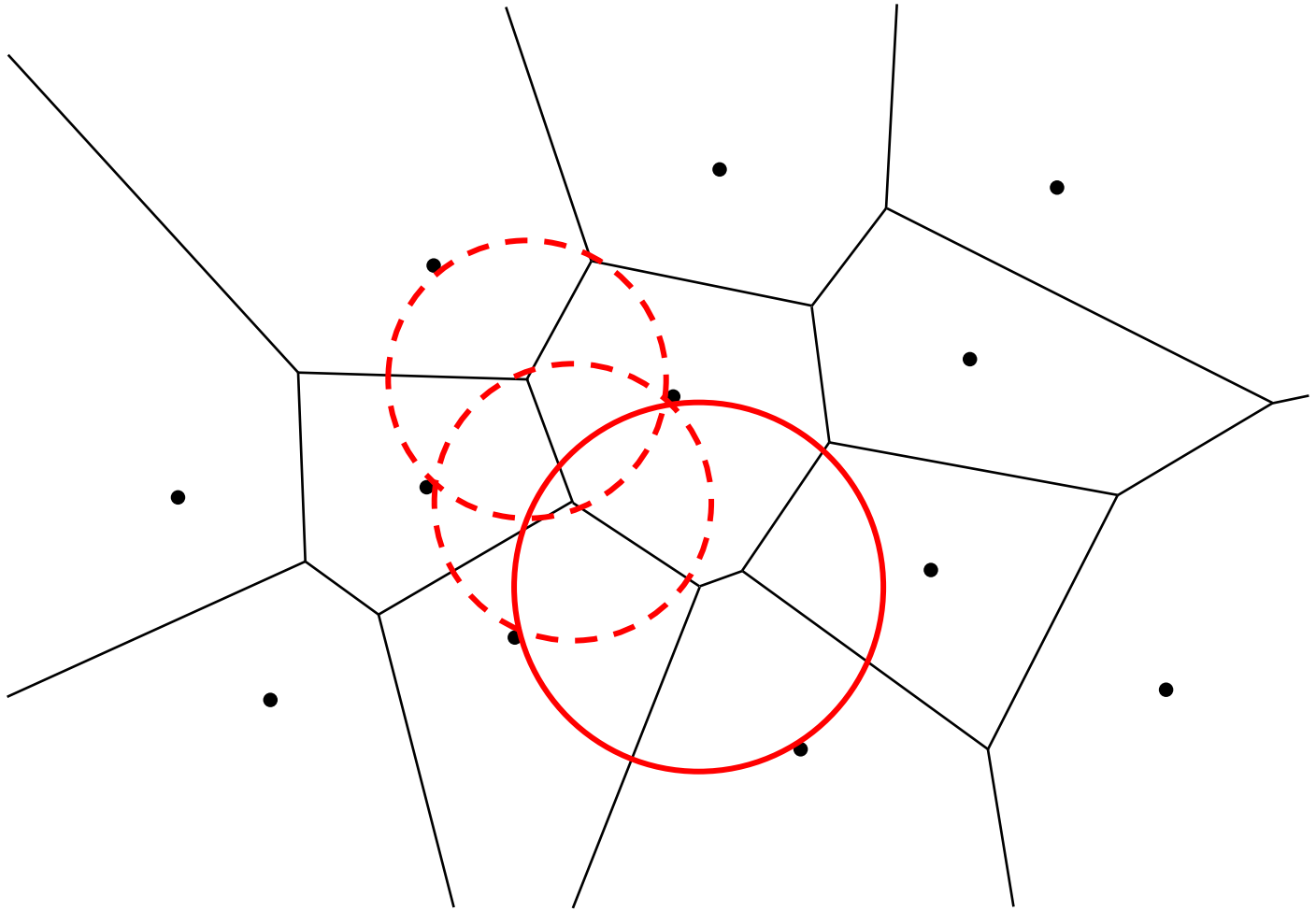
↓

Hamiltonian Circuit

$$d(H) \leq 1.5d(H^*)$$

H = Algorithm Solution
H^*= Optimal Solution

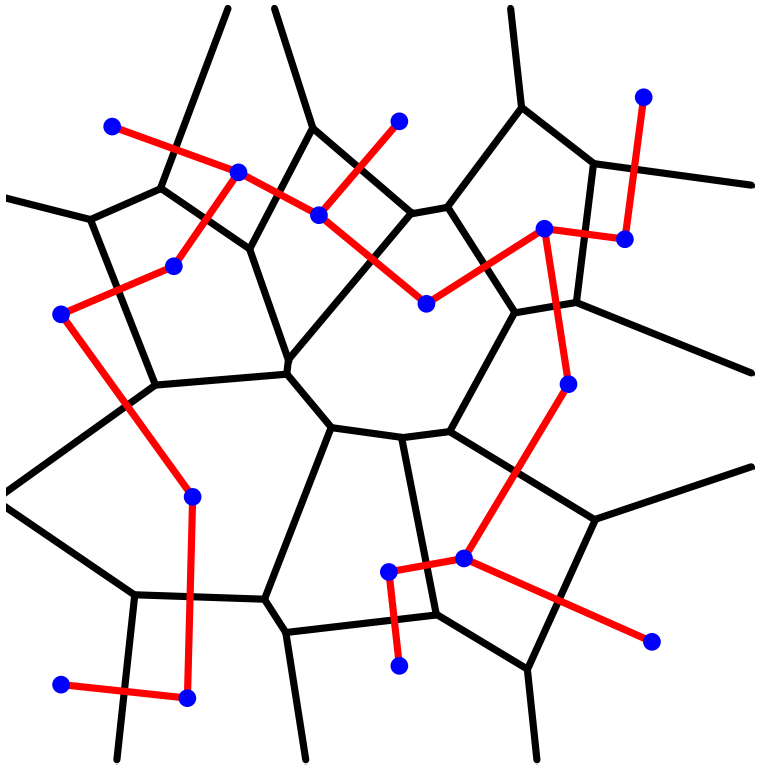
Voronoi Diagram



- Voronoi diagram is a tessellation based on distance to points in a specific subset of the plane.
- Geometric structure with useful geometric properties

Easily Solved Problems Using VD(P)

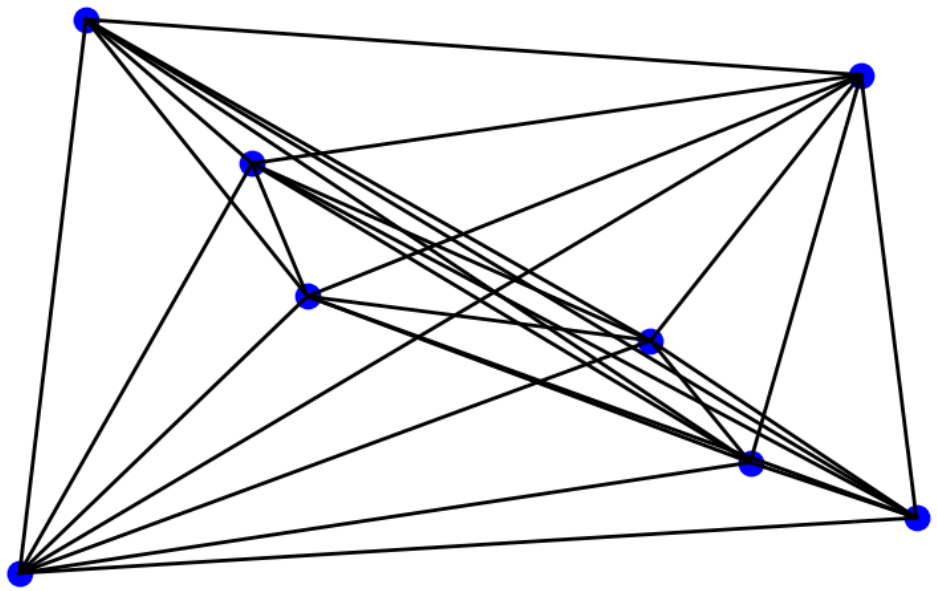
- Convex hull
- Largest empty circle
- Delaunay triangulation
- Minimum spanning tree
- Nearest neighbor
- Closest pair
- Etc.



Minimum Spanning Tree

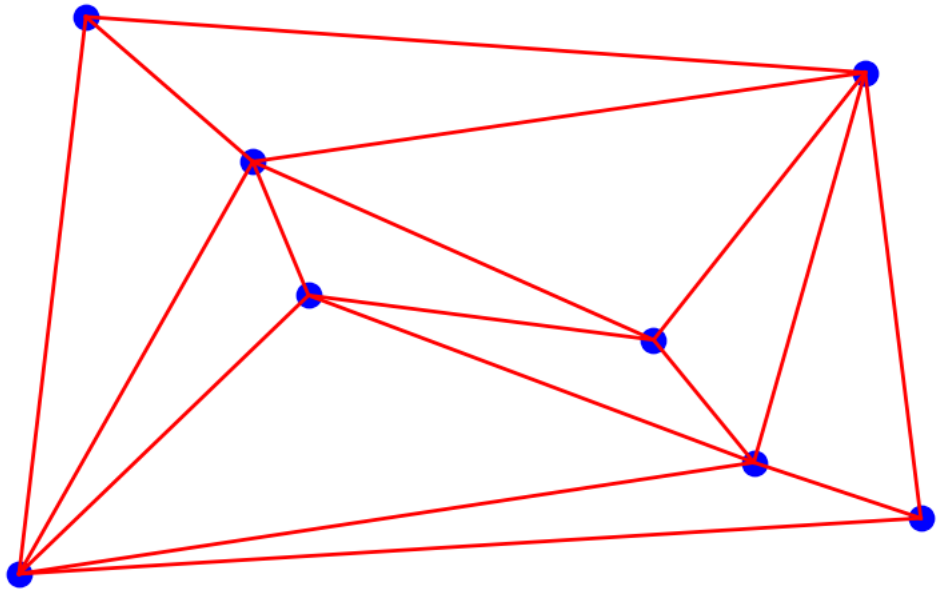
그래프 내의 모든 Vertex를 포함하는 Tree

Complete Graph



#Edges : $\frac{n(n-1)}{2}$

Delaunay Triangulation

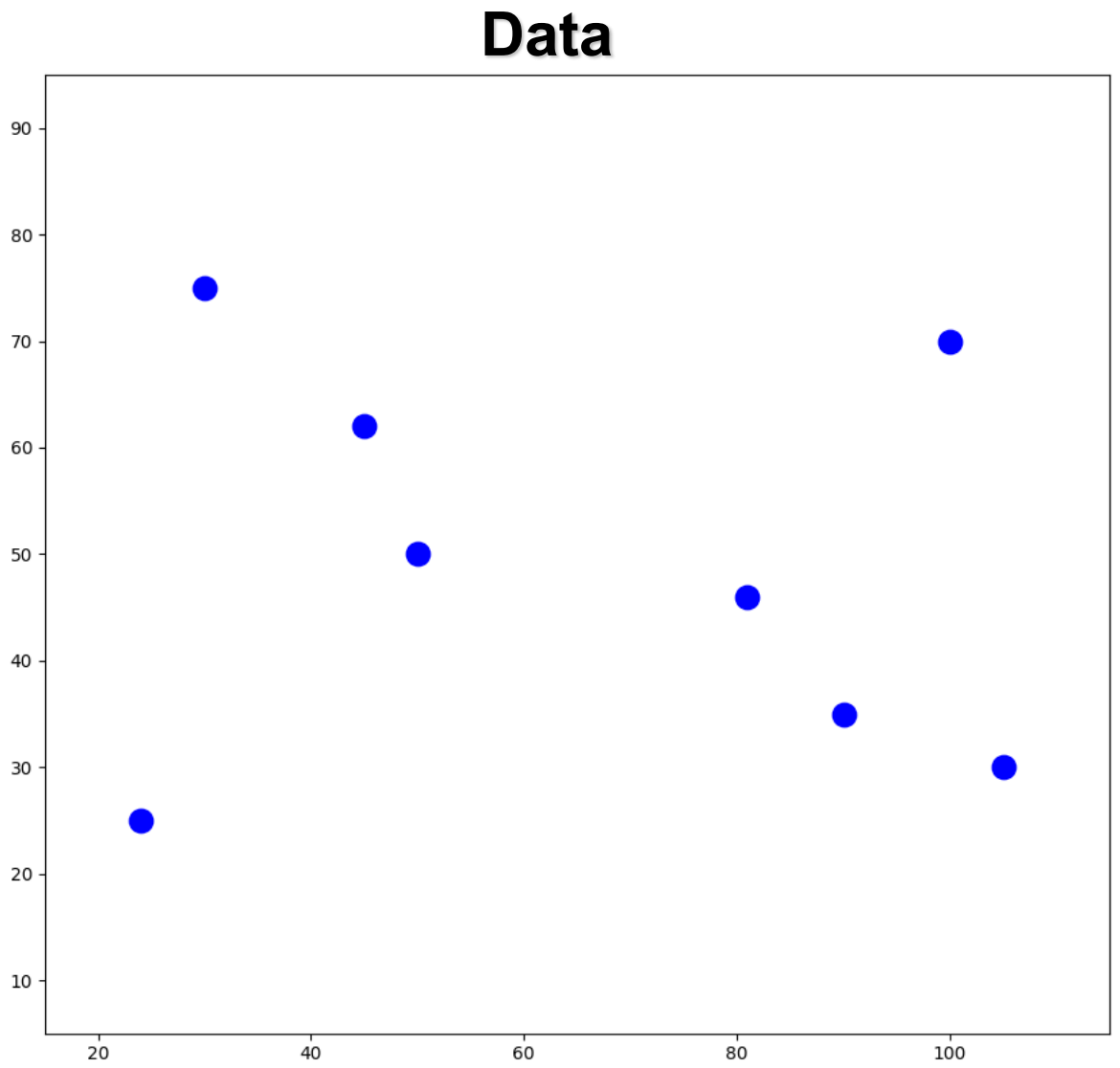


#Edges : $3n - 3 - k$

k = Convex Hull Point

2-Approximation Algorithm (1/9)

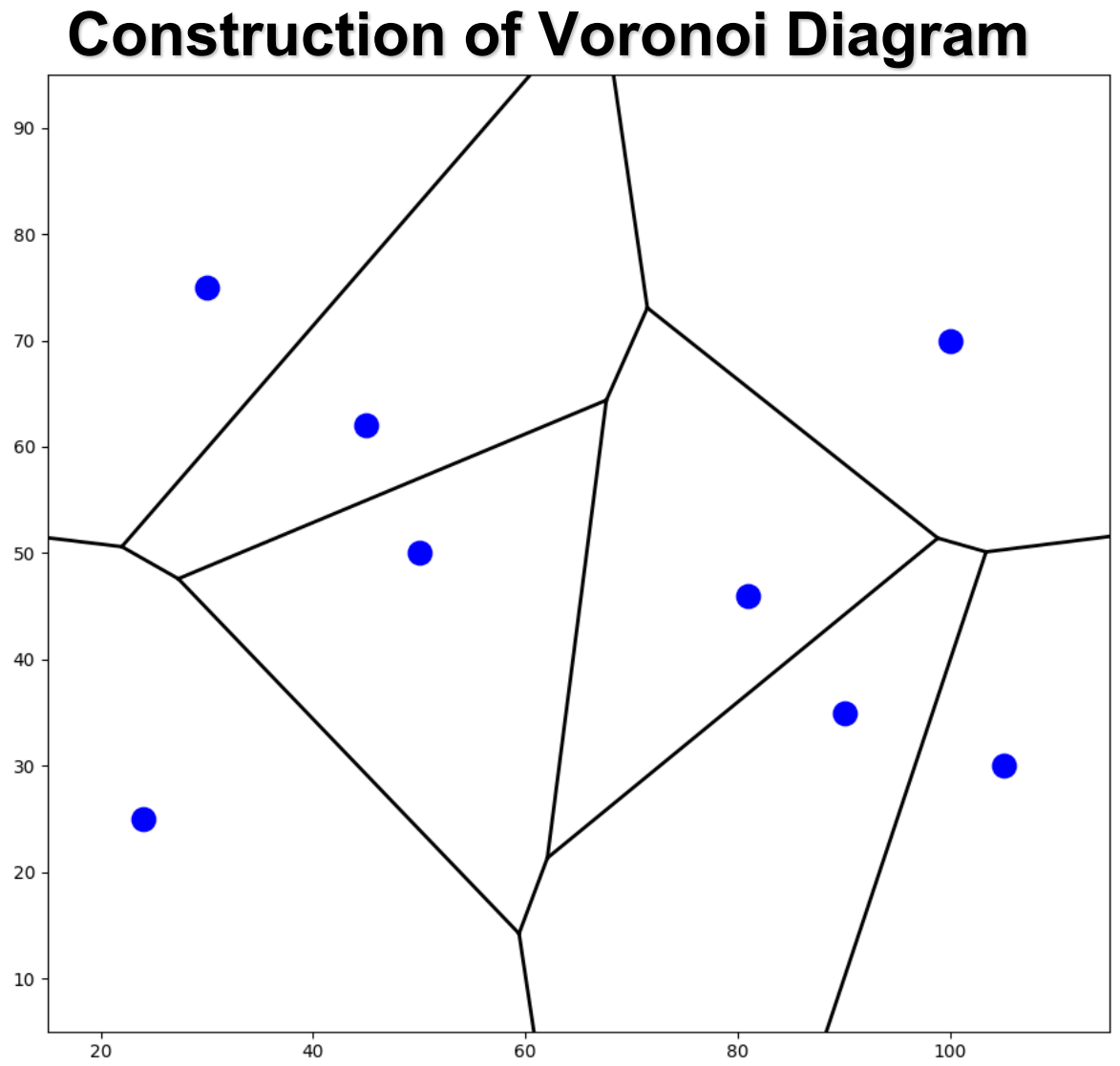
- Data



2-Approximation Algorithm (2/9)

Data

VD Edges



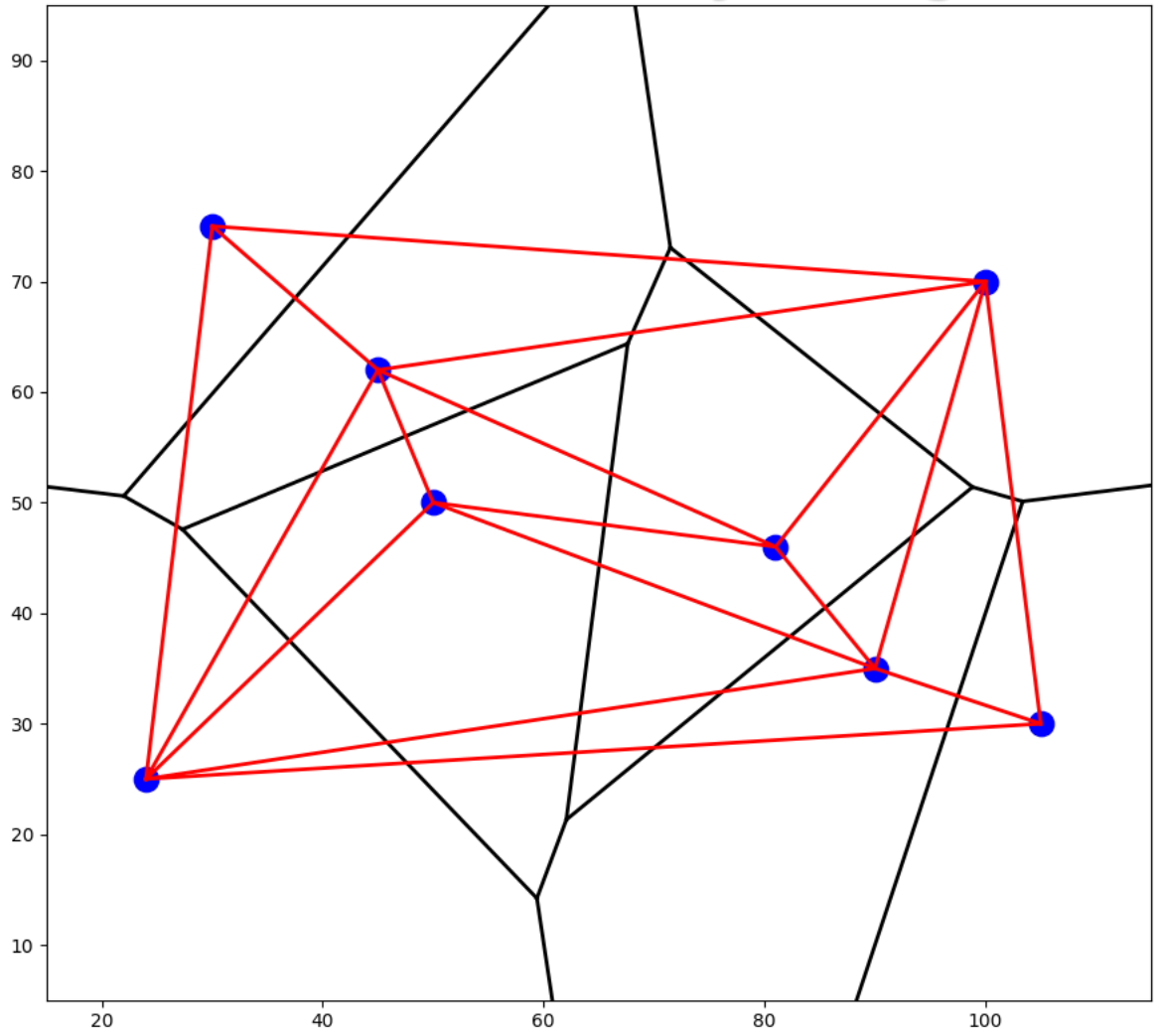
2-Approximation Algorithm (3/9)

Construction of Delaunay Triangulation

Data

VD Edges

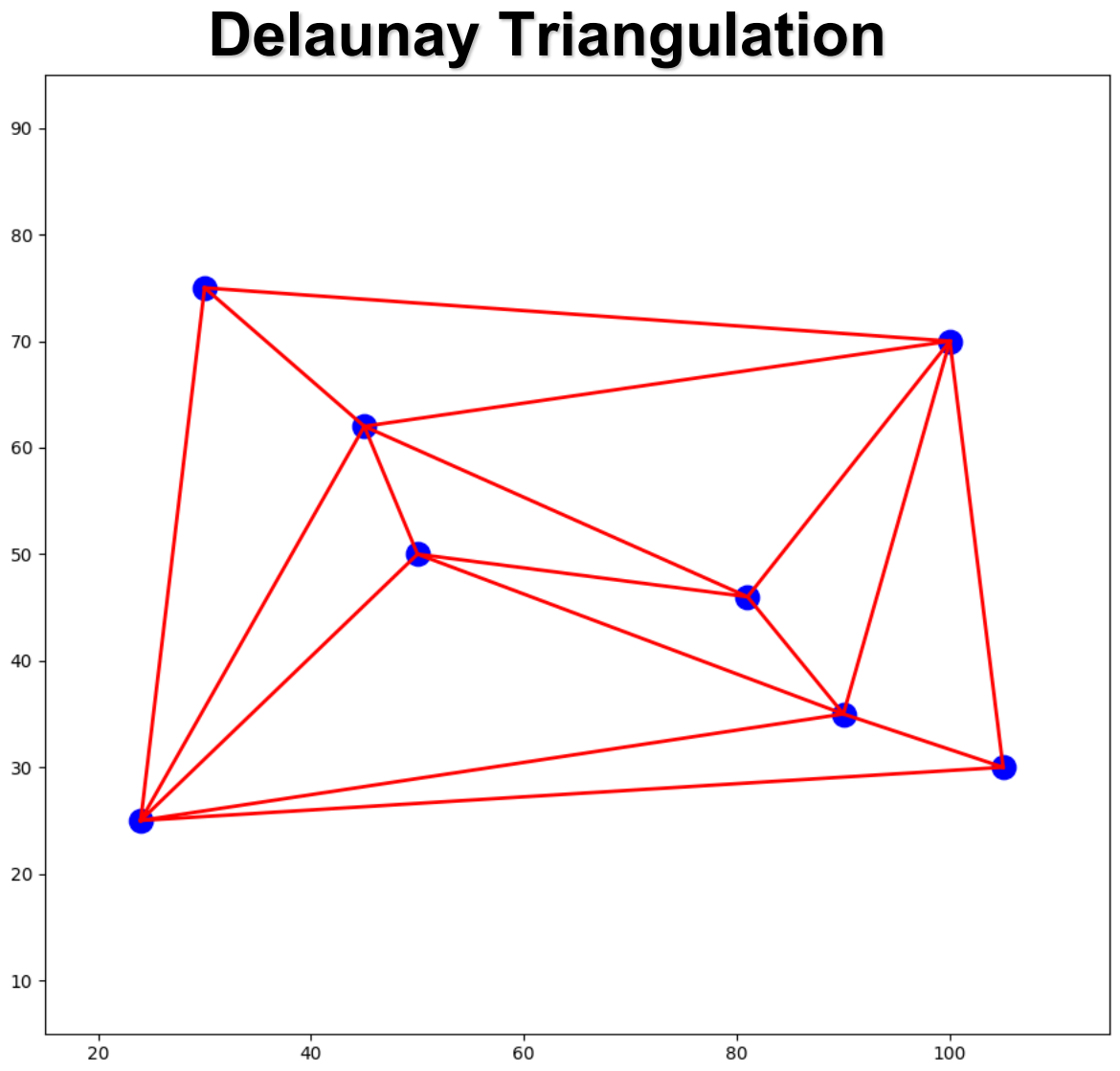
Delaunay Triangulation



2-Approximation Algorithm (4/9)

Data

Delaunay Triangulation

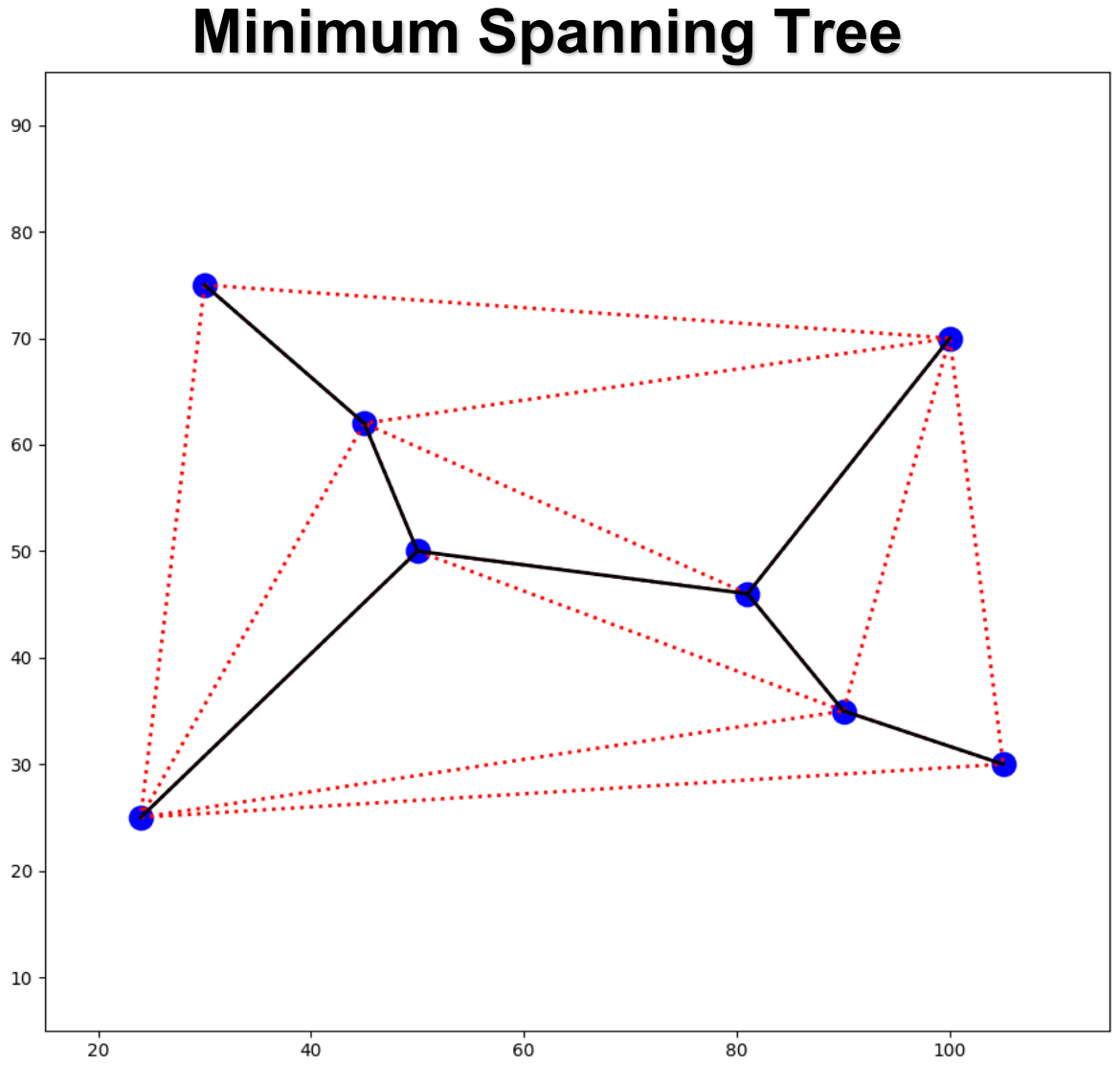


2-Approximation Algorithm (5/9)

Data

Delaunay Triangulation

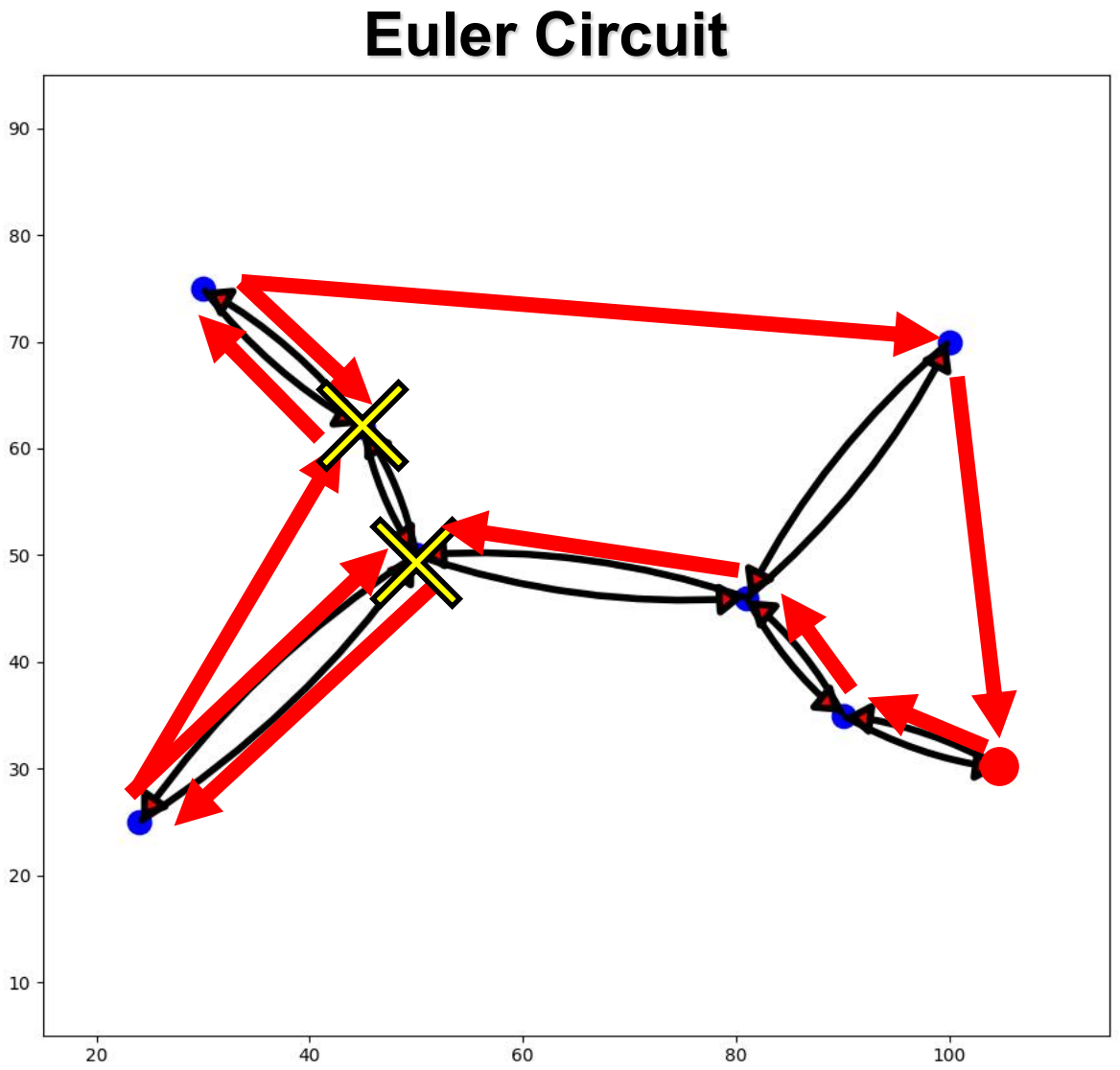
Minimum Spanning Tree



2-Approximation Algorithm (6/9)

Data

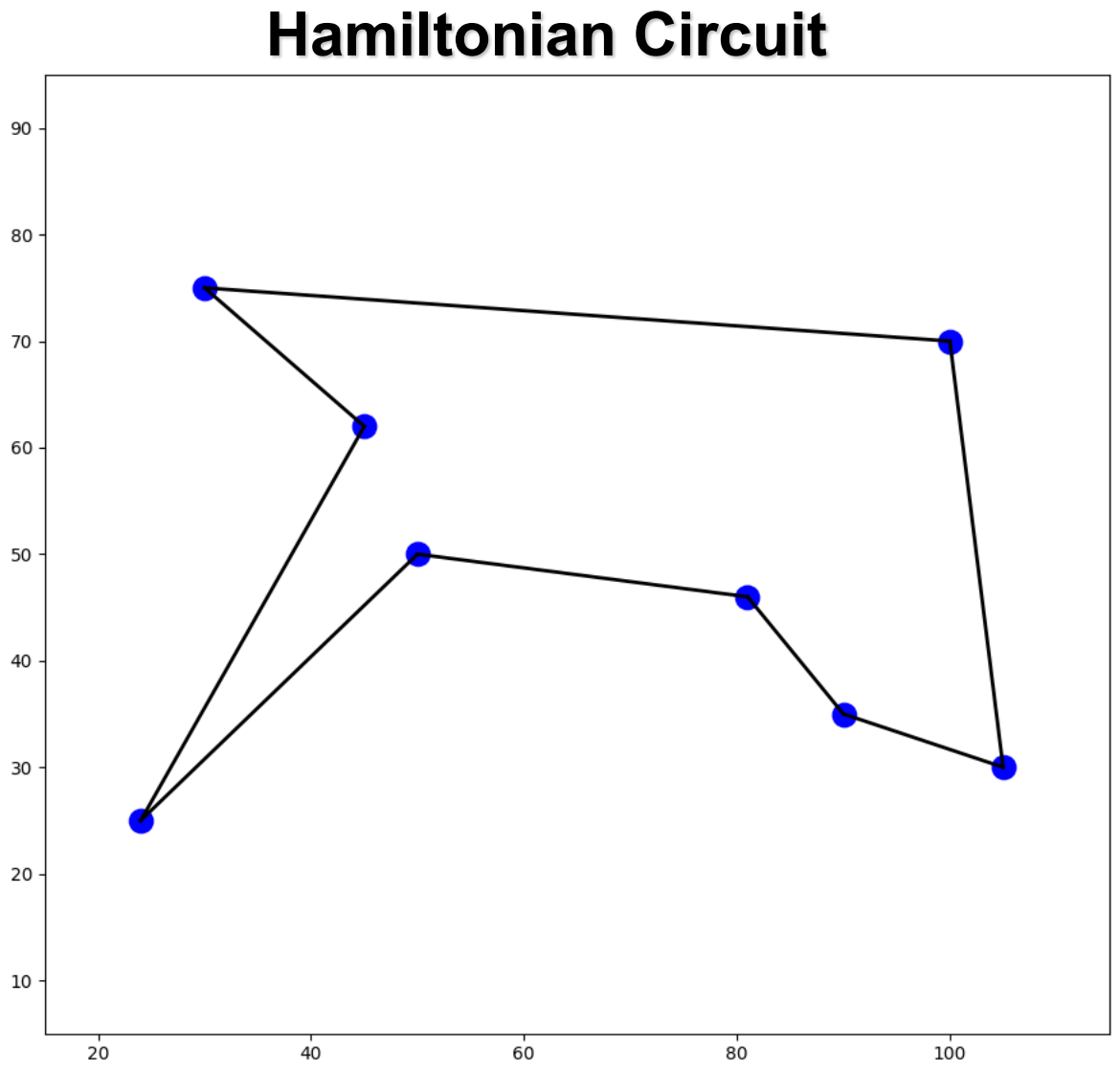
Euler Circuit



2-Approximation Algorithm (7/9)

Data

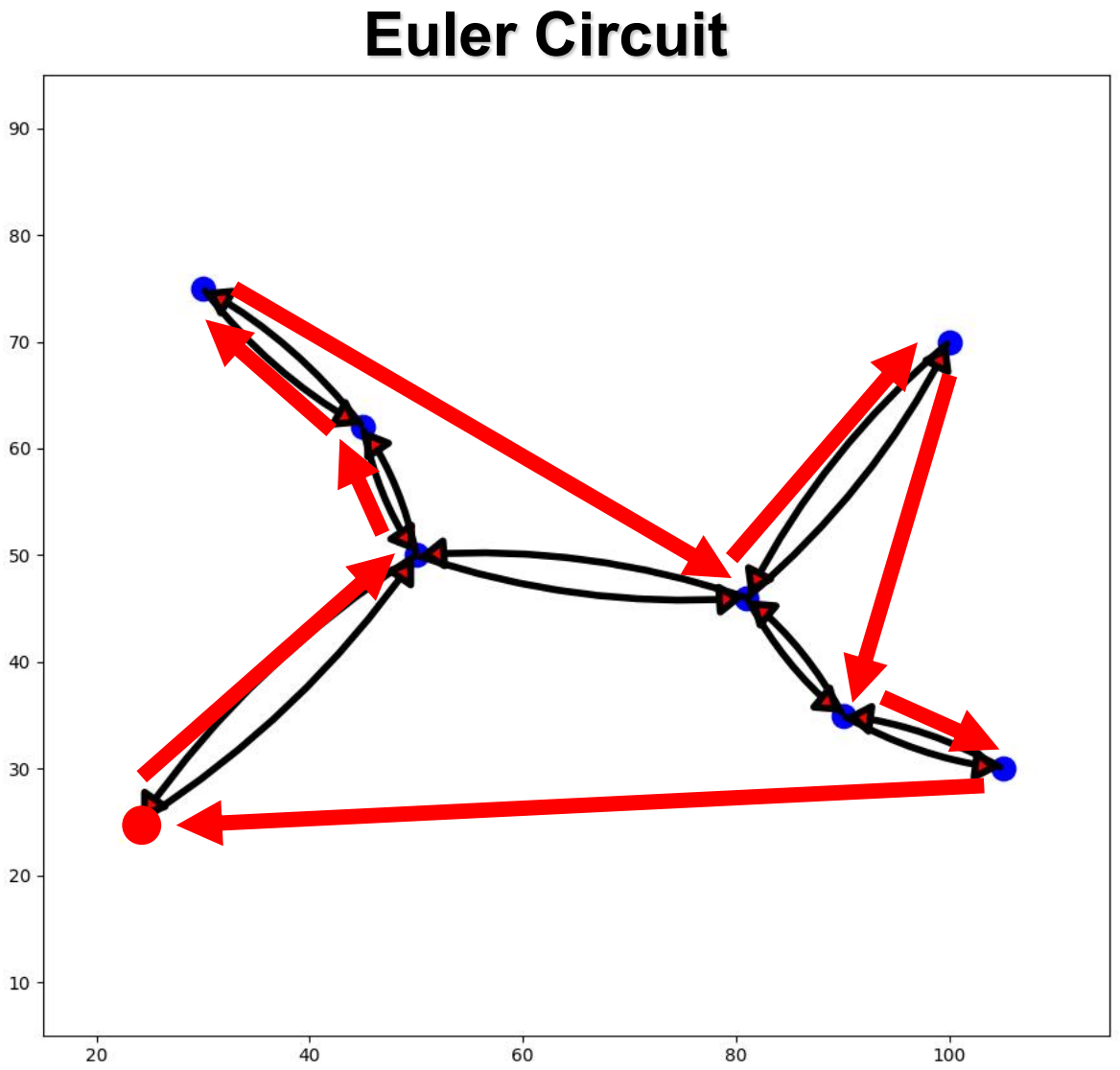
Hamiltonian Circuit



2-Approximation Algorithm (8/9)

Data

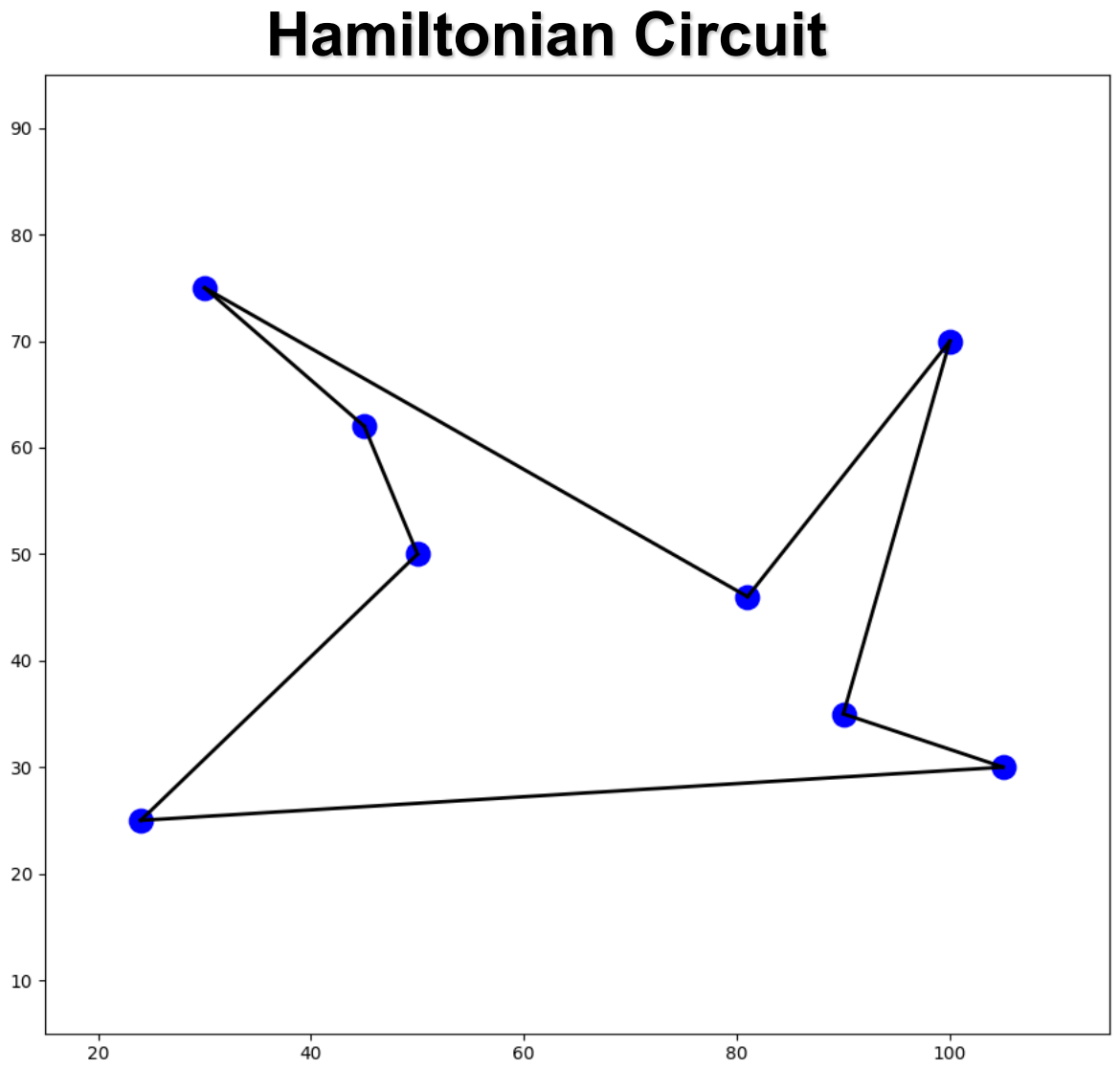
Euler Circuit



2-Approximation Algorithm (9/9)

Data

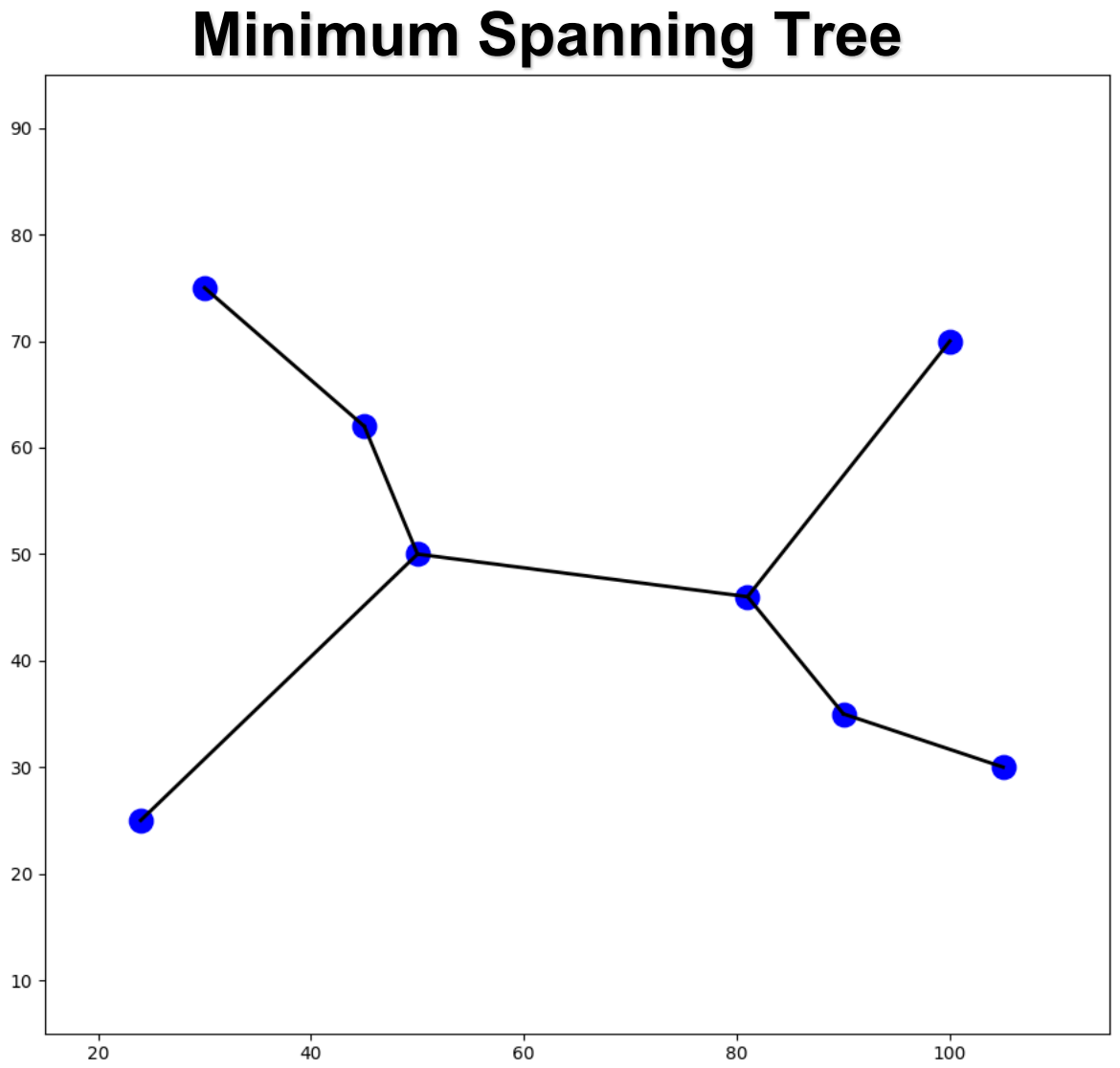
Hamiltonian Circuit



1.5-Approximation Algorithm (1/5)

Data

Minimum Spanning Tree

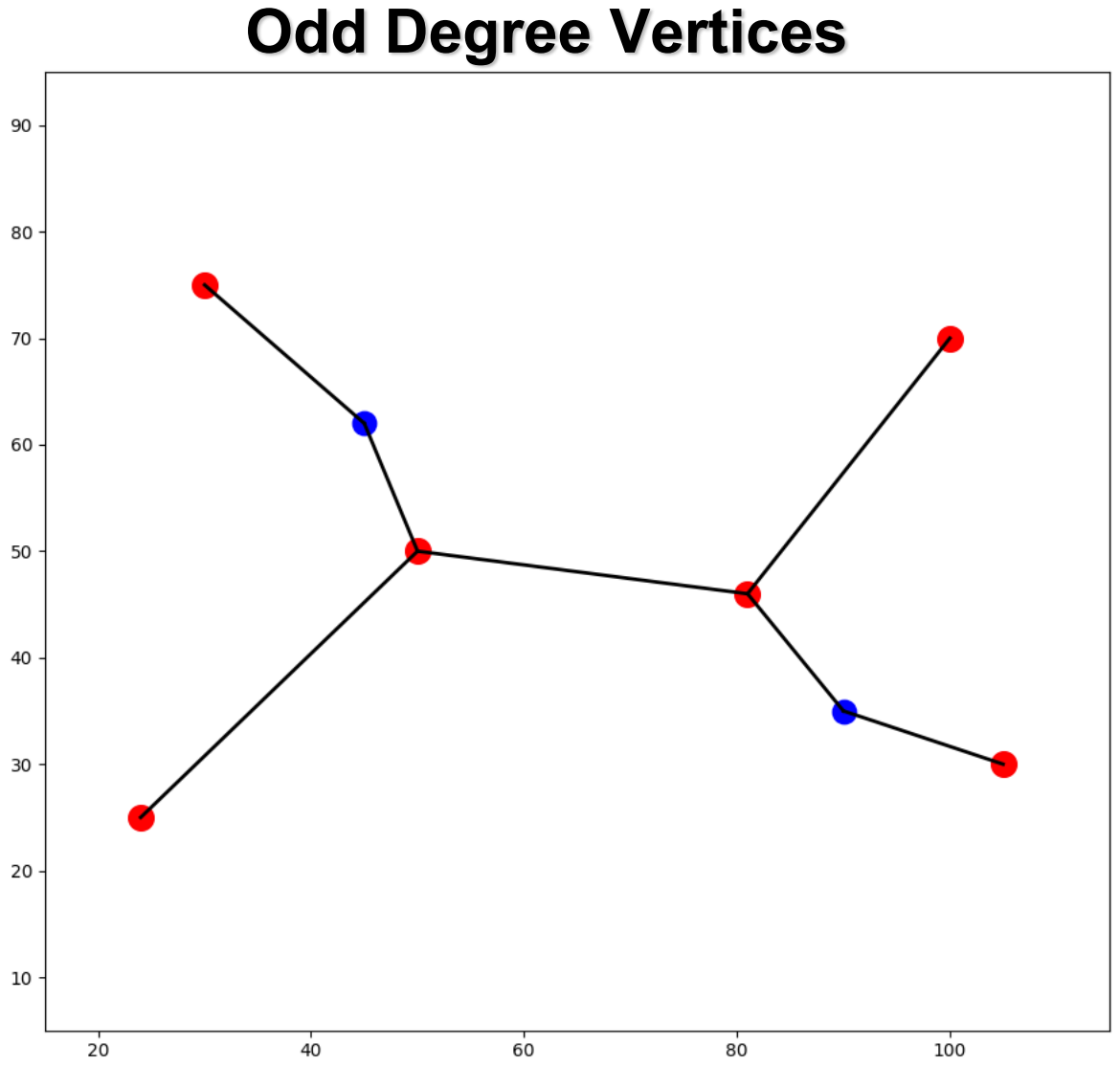


1.5-Approximation Algorithm (2/5)

Data

Odd Degrees Data

Minimum Spanning Tree



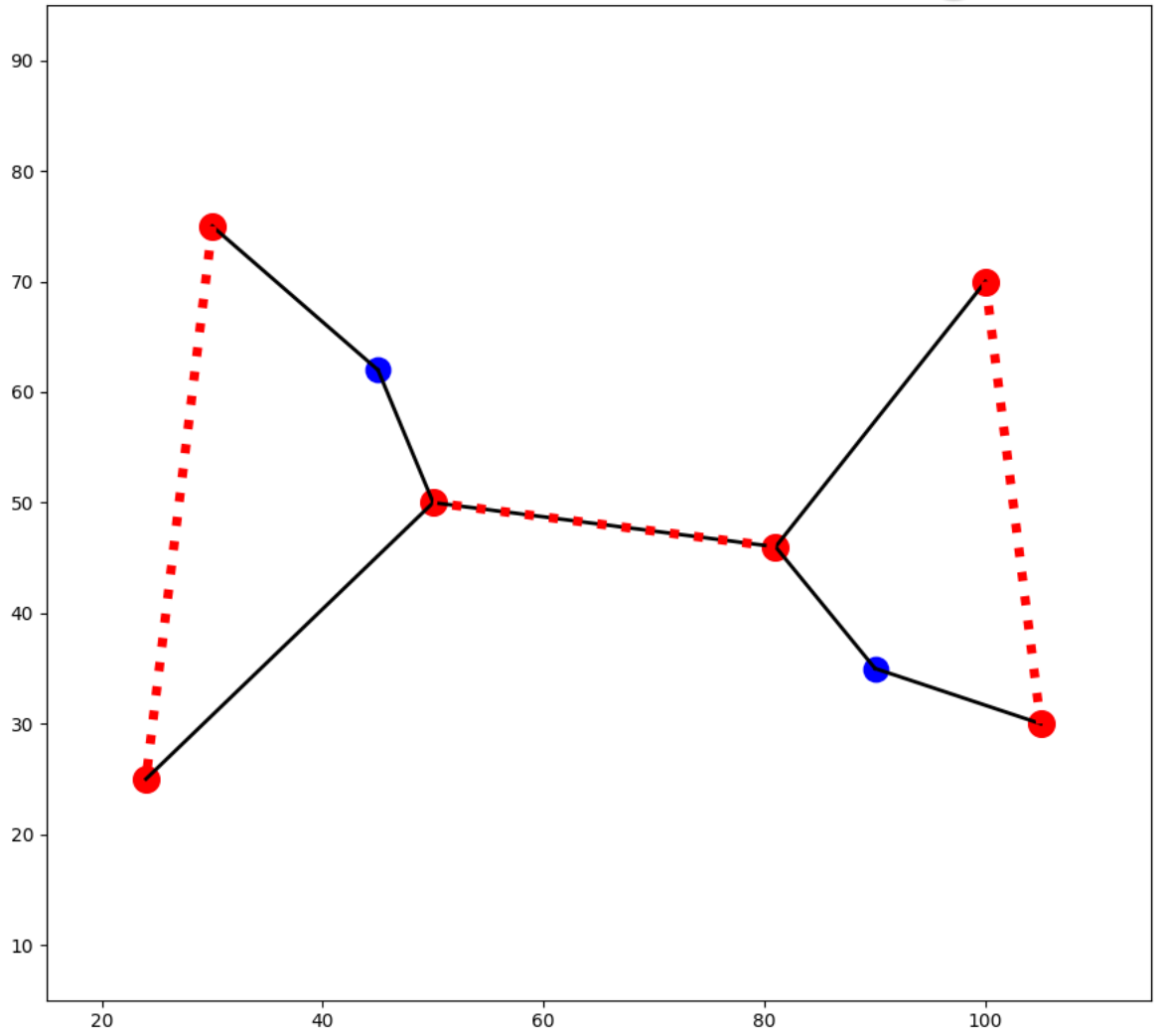
1.5-Approximation Algorithm (3/5)

Data

Odd Degrees Data

Minimum Spanning Tree

Minimum Perfect Matching

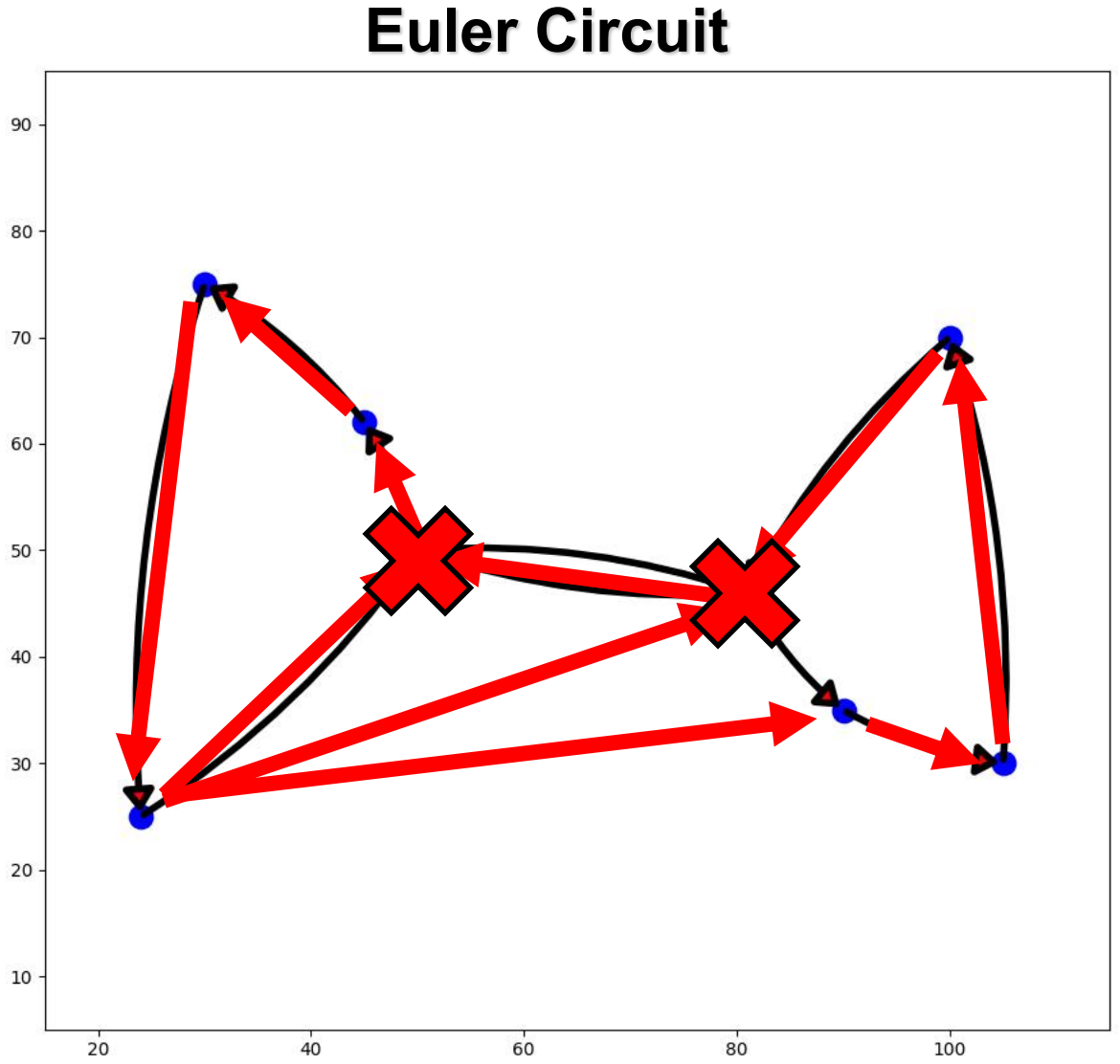


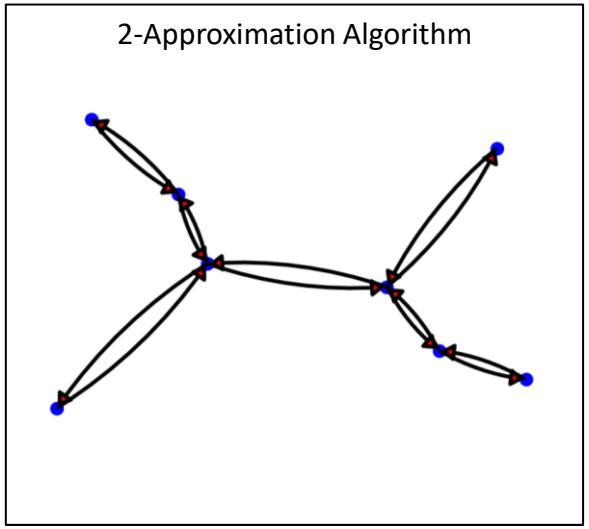
1. Matching
   - 서로 만나지 않는 Edge들의 집합
2. Perfect Matching
   - 모든 Vertex를 포함하는 Matching
3. Minimum Perfect Matching
   - Perfect Matching 중 Edge의 길이의 합이 가장 긴 조합

1.5-Approximation Algorithm (4/5)

Data

Euler Circuit

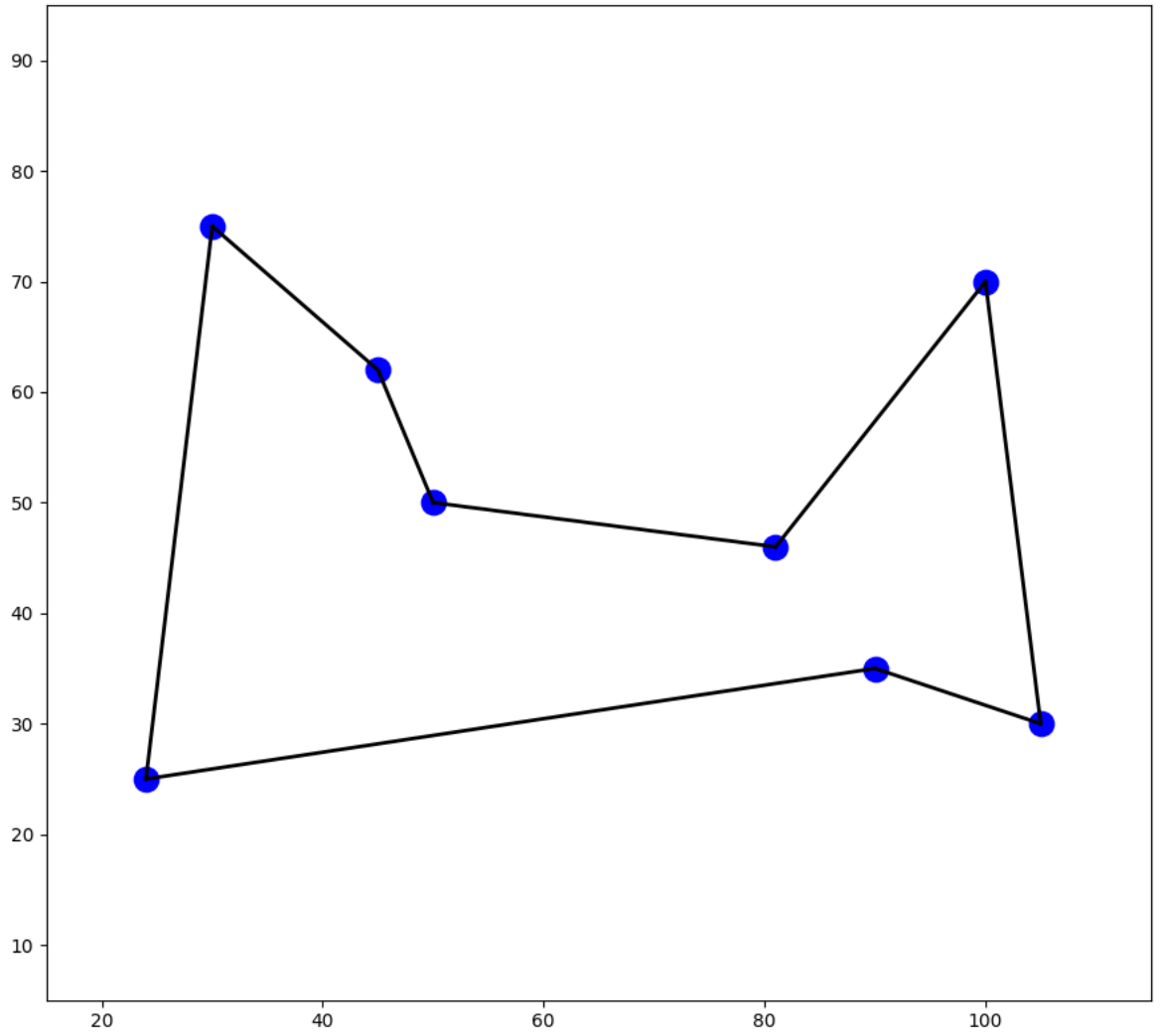


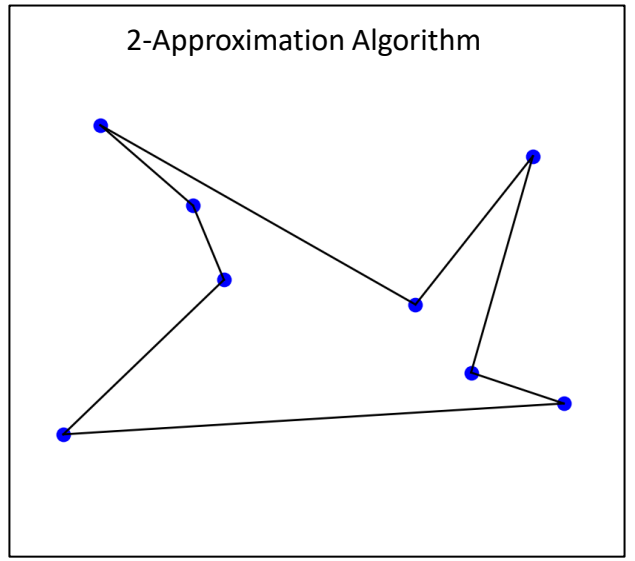


1.5-Approximation Algorithm (5/5)

Data

Hamiltonian Circuit

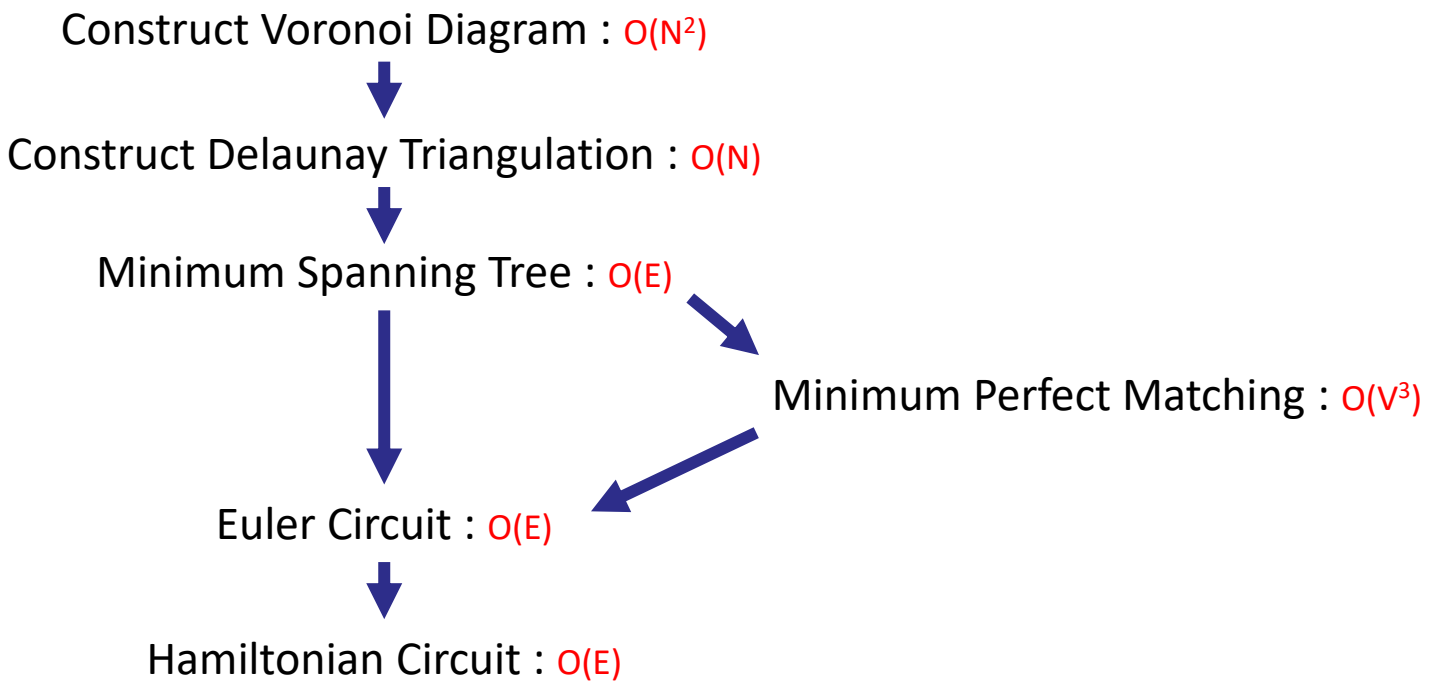




Time Complexity

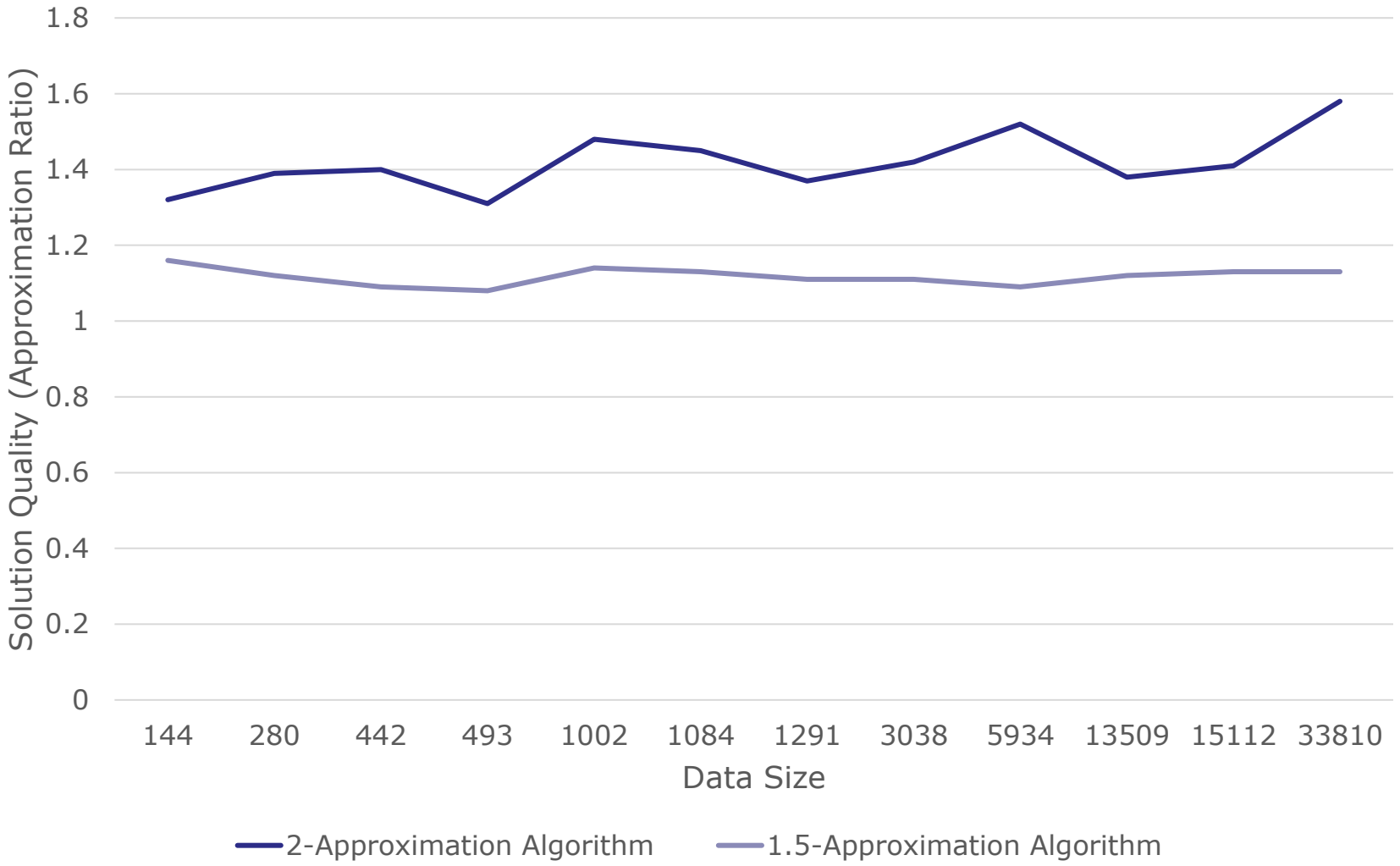
2-Approximation Algorithm

1.5-Approximation Algorithm
(Christofides Algorithm)

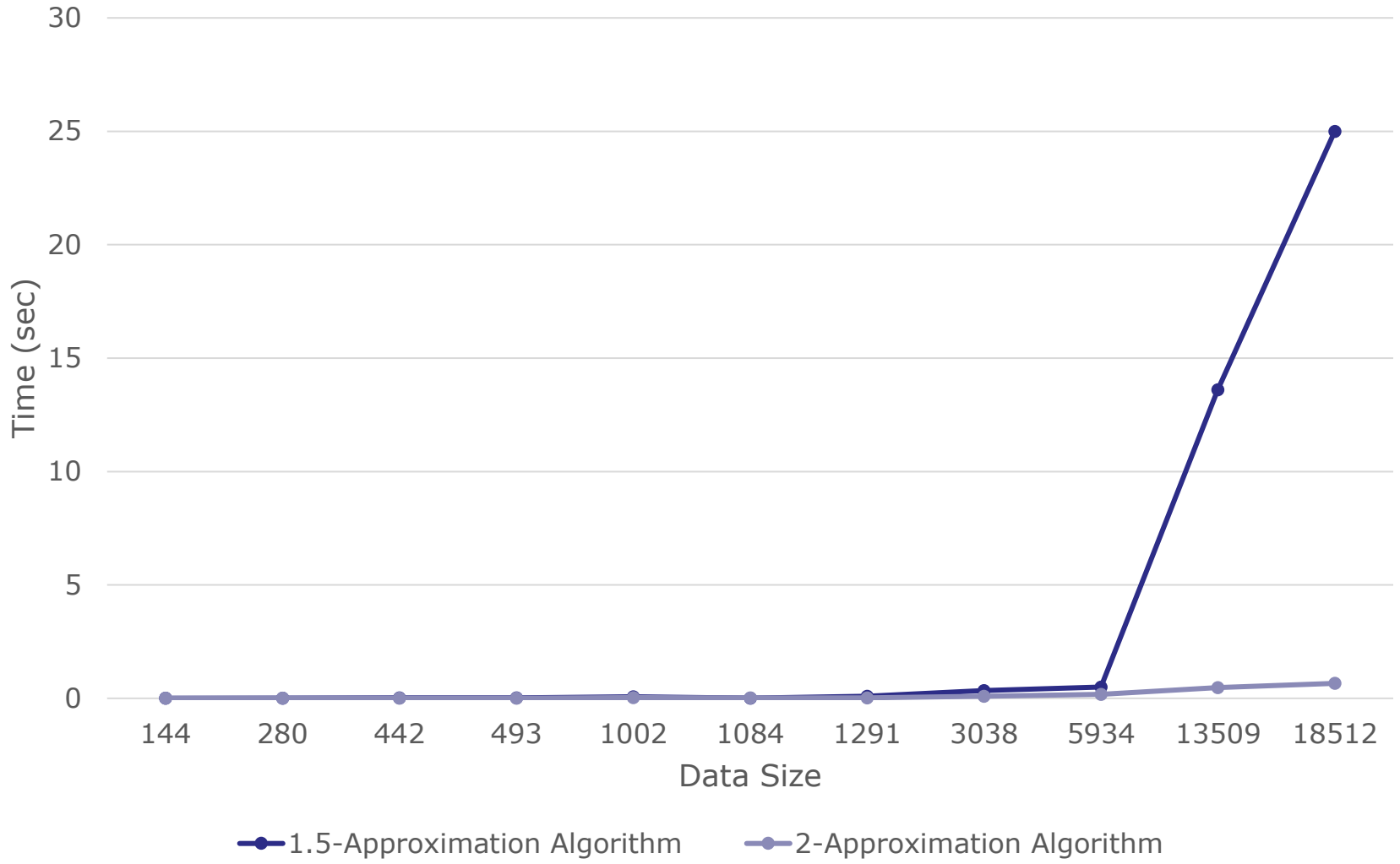


Time Complexity is always worst case

Compare Algorithms (Solution)

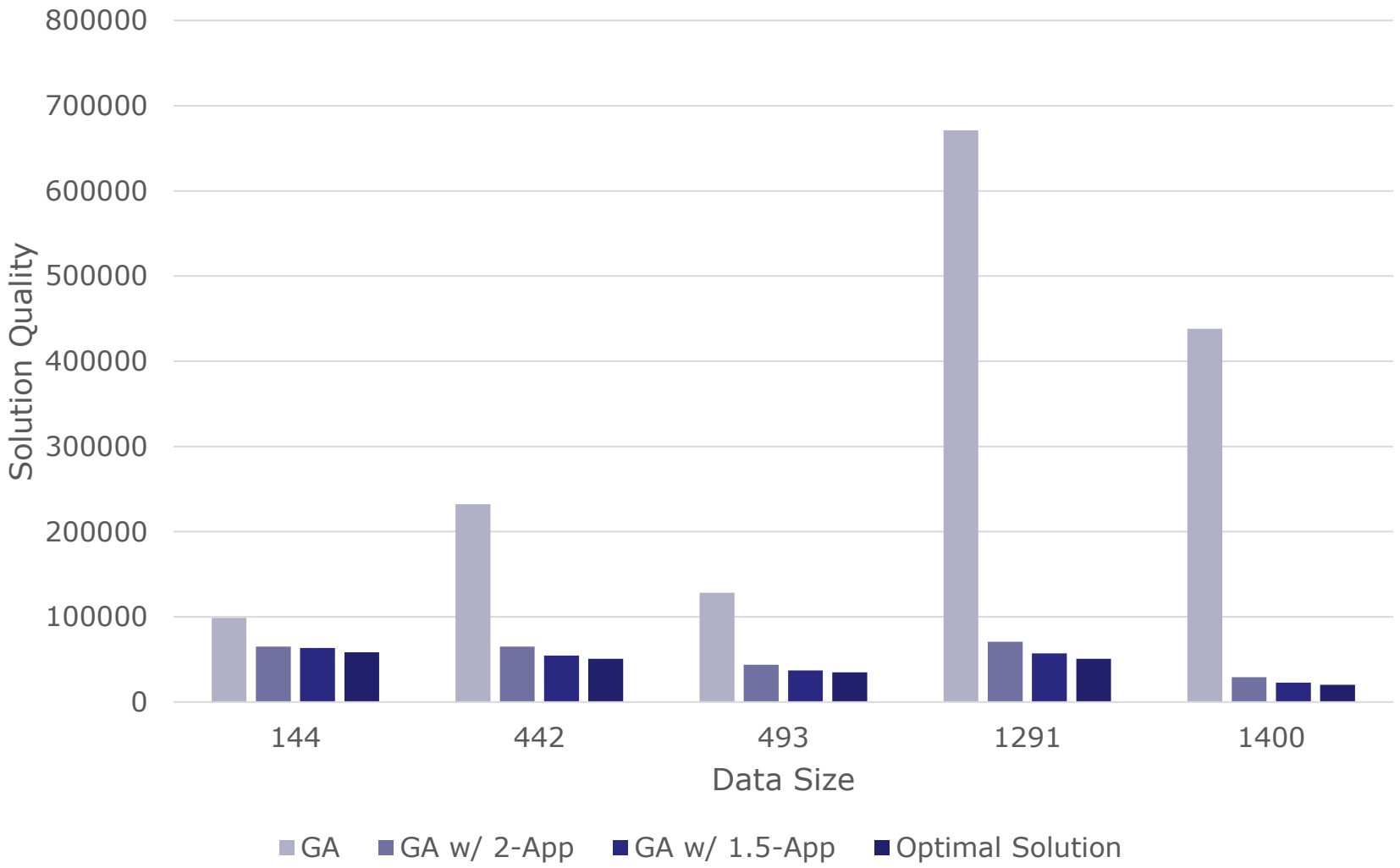


Compare Algorithms (Time)



Genetic Algorithm

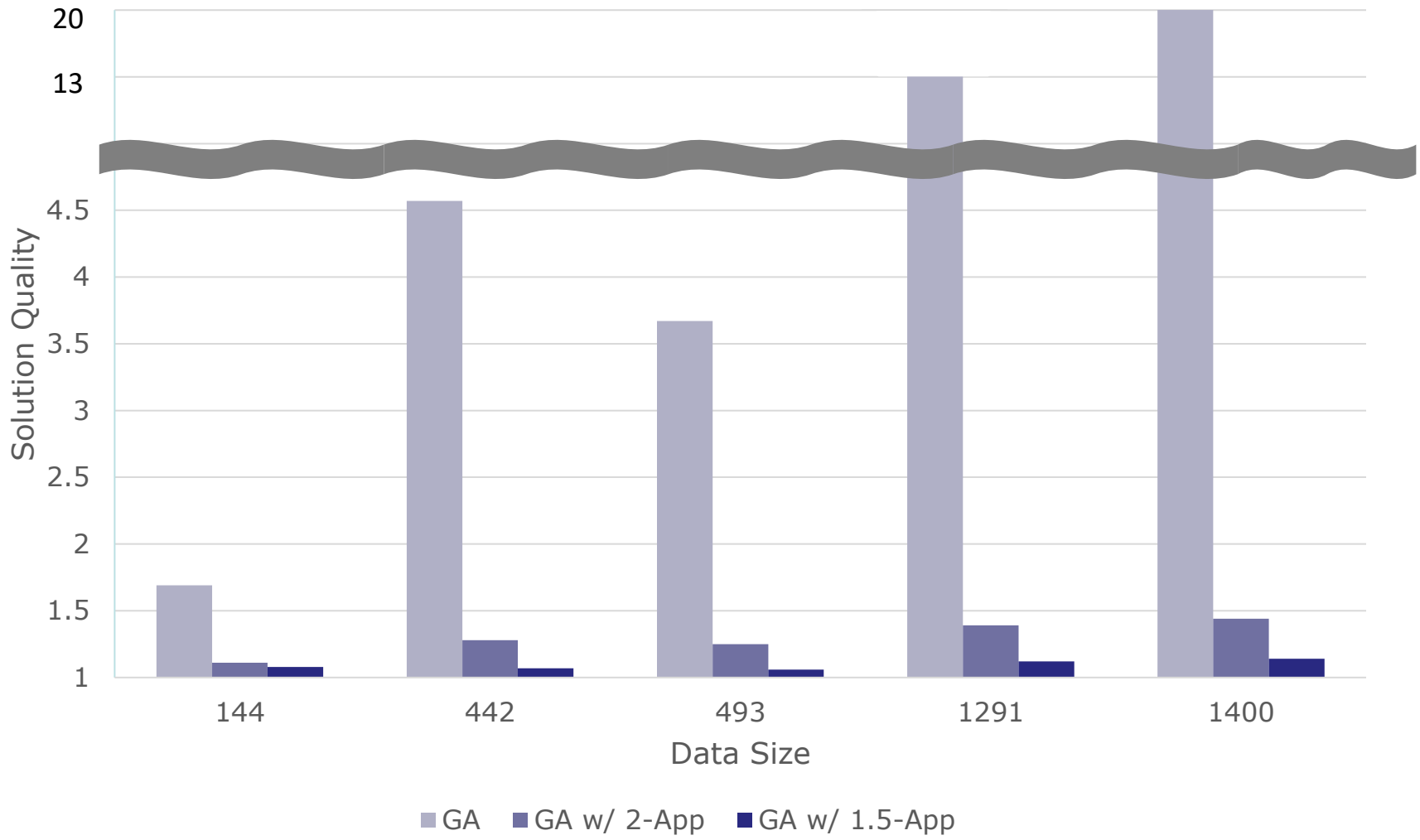
Generation : 1000



Source : TSPLIB

Genetic Algorithm

Generation : 1000



Source : TSPLIB

Conclusion

1. 분석

   - GA를 포함한 전체 계산시간에서 Approximation Algorithm은 영향이 적음.
     그러므로, 1.5-Approximation Algorithm을 사용하는 것이 일반적으로 더 좋음.

2. 한계
   - Approximation Algorithm의 Solution이 cross되지 않는다는 것을 보장해주지 않음.
   - 기존의 GA Operators는 계산 속도가 빠르지 않음.

3. 개선점
   - Cross되는 Edge를 Swap하여 좋은 Initial Solution을 구함.
   - GA Operators를 개선.

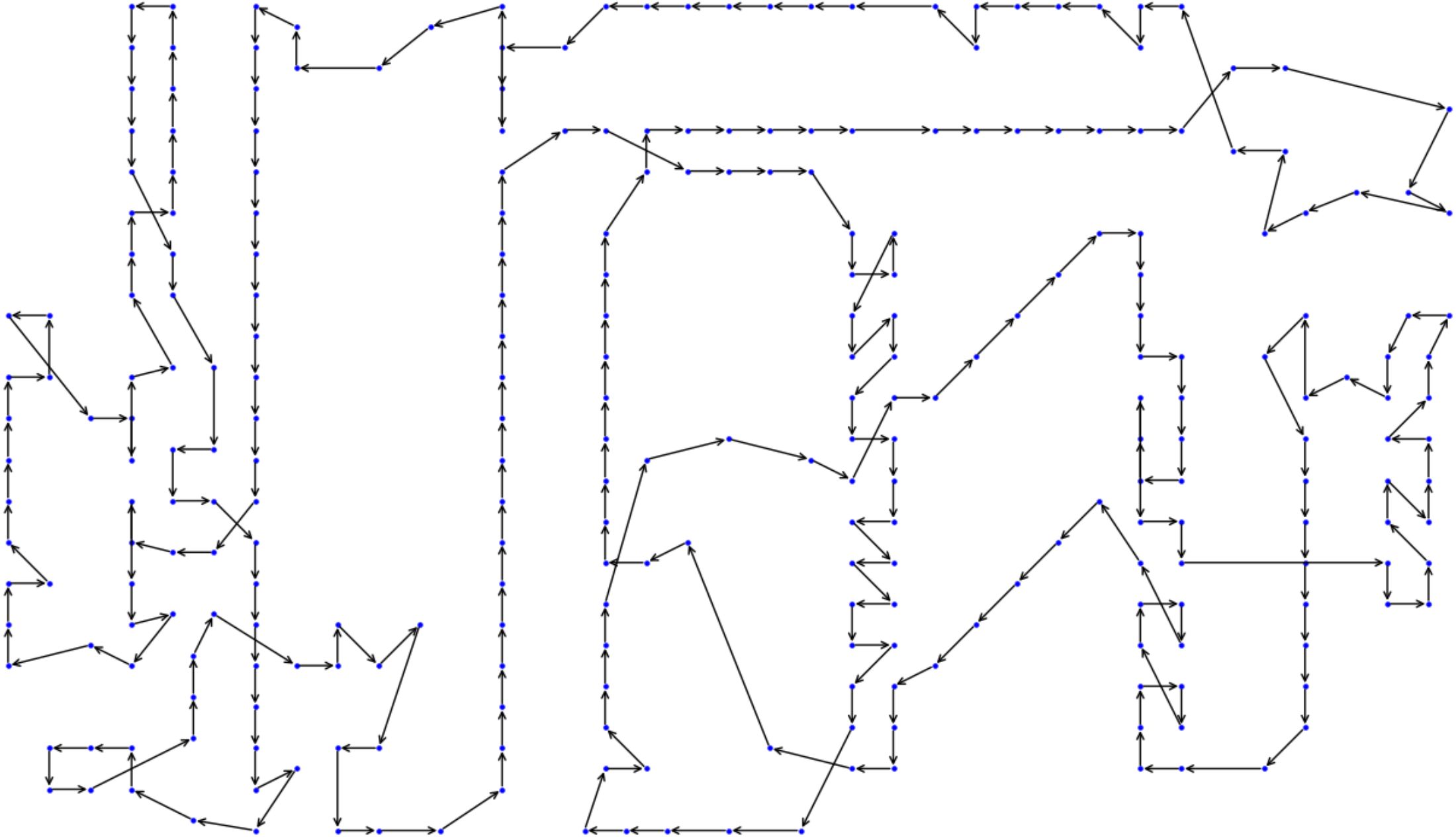
References

1. N. Christofides, *Worst-case analysis of a new heuristic for the traveling salesman problem*, Report 388, Graduate School of Industrial Administration, Carnegie Mellon University, 1976.

2. T. Cormen, C. Leiserson, and R. Rivest, *Introduction to Algorithms*, The MIT Press, 1990.

3. J. Edmonds, “Paths Trees and Flowers”, *Canadian Journal of Math*, vol 17 (1965), 449-467.

감사합니다



Approximation Algorithm

2-Approximation Algorithm

$$d(H^*) \geq d(H^* - e) \geq d(T)$$

$$\downarrow$$

$$d(C) = 2d(T)$$

$$\downarrow$$

$$d(H) \leq d(C)$$

$$\downarrow$$

$$d(H) \leq 2d(T)$$

$$\downarrow$$

$$d(H) \leq 2d(H^*)$$

1.5-Approximation Algorithm
(Christofides Algorithm)

$$d(H^*) \geq d(H^* - e) \geq d(T)$$

$$\downarrow$$

$$d(C) = d(T) + d(J)$$

$$\downarrow$$

$$d(J_1) + d(J_2) \leq d(H^*)$$

$$\downarrow$$

$$d(J) \leq \min\{d(J_1), d(J_2)\} \leq 0.5d(H^*)$$

$$\downarrow$$

$$d(H) \leq 1.5d(H^*)$$

$d(F)$= Sum of the distances of the Edges
H = Approximation Algorithm Circuit
H^*=Optimal Circuit
T = Minimum Spanning Tree
J= Minimum Perfect Matching

1.5-Approximation Algorithm

	Voronoi Diagram	Delaunay Triangulation	Minimum Spanning Tree	Minimum Perfect Matching	Euler & Hamiltonian	Total Time	Fitness Value	Optimal Solution	
pr144	0.002 s	0.0004 s	0.0002 s	0.0007 s	0.0008 s	0.004 s	68078.7	58537	1.16x
a280	0.004 s	0.0008 s	0.0004 s	0.003 s	0.001 s	0.009 s	2904.12	2579	1.12x
pcb442	0.006 s	0.001 s	0.0008 s	0.009 s	0.002 s	0.018 s	55373.3	50778	1.09x
d493	0.007 s	0.002 s	0.001 s	0.01 s	0.002 s	0.022 s	38060.5	35002	1.08x
vm1084	0.014 s	0.004 s	0.002 s	0.03 s	0.005 s	0.012 s	274864	239297	1.14x
d1291	0.02 s	0.005 s	0.002 s	0.01 s	0.005 s	0.087 s	57732.6	50801	1.13x
pr1002	0.02 s	0.006 s	0.003 s	0.03 s	0.005 s	0.064 s	289057	259045	1.11x
pcb3038	0.06 s	0.014 s	0.007 s	0.25 s	0.01 s	0.341 s	153866	137694	1.11x
rl5934	0.09 s	0.024 s	0.016 s	0.16 s	0.02 s	0.35 s	607409	556045	1.09x
usa13509	0.3 s	0.07 s	0.05 s	13.1 s	0.07 s	13.6 s	22361000	19982859	1.12x
d15112	0.24 s	0.07 s	0.06 s	20.8 s	0.07 s	21.2 s	1773420	1573084	1.13x
pla33810	0.78 s	0.16 s	0.12 s	22.5 s	0.17 s	23.7 s	74709400	66048945	1.13x

평균 : 1.12x

Source : TSPLIB

2-Approximation Algorithm

	Voronoi Diagram	Delaunay Triangulation	Minimum Spanning Tree	Euler & Hamiltonian	Total Time	Fitness Value	Optimal Solution	
pr144	0.002 s	0.0008 s	0.0004 s	0.001 s	0.004 s	77494.2	58537	1.32x
a280	0.003 s	0.0008 s	0.0004 s	0.002 s	0.006 s	3591.94	2579	1.39x
pcb442	0.006 s	0.002 s	0.001 s	0.003 s	0.012 s	71157.9	50778	1.4x
d493	0.006 s	0.002 s	0.001 s	0.003 s	0.012 s	45808.1	35002	1.31x
vm1084	0.014 s	0.004 s	0.002 s	0.005 s	0.025 s	353968	239297	1.48x
d1291	0.01 s	0.004 s	0.002 s	0.007 s	0.023 s	73682	50801	1.45x
pr1002	0.02 s	0.007 s	0.003 s	0.007 s	0.037 s	353794	259045	1.37x
pcb3038	0.06 s	0.011 s	0.007 s	0.02 s	0.098 s	195772	137694	1.42x
rl5934	0.09 s	0.024 s	0.018 s	0.04 s	0.172 s	844207	556045	1.52x
usa13509	0.26 s	0.07 s	0.05 s	0.1 s	0.48 s	27639800	19982859	1.38x
d15112	0.2 s	0.06 s	0.05 s	0.1 s	0.41 s	2216250	1573084	1.41x
pla33810	0.8 s	0.15 s	0.11 s	0.2 s	1.26 s	104029000	66048945	1.58x

평균 : 1.42x

Source : TSPLIB